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The Effect of Dispersal Corridor on the Permanence of Competitive Ecosystems

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(Abstract) The theoretical sufficient conditions for the permanence of discrete time models for competitive ecosystems with two species in patchy environment without or with dispersal corridor are obtained. Based on these sufficient conditions, the effect of dispersal corridor on the permanence of the competitive ecosystems is analyzed. Numerical simulation is given, which confirms the theoretical results.

Keywords: Competitive Ecosystem; Dispersal Corridor; Permanence; Numerical Simulation

1. INTRODUCTION

Due to spatial heterogeneity and the spread of human activities, some habitats of biological species have been separated in isolated patches [1]. Dispersal corridors are established among these patches in order to meet the needs of certain species for resources, escaping for the predators or other competitors, exchanging the gene flow and other aspects to maintain the biodiversity [2-4]. At least, one purpose of the dispersal corridors is to avoid the extinction of the biological species in these ecosystems [5]. There are studies on the permanence or extinction of the biological species in patchy environment via theoretical analysis of mathematical models [1-3,5-7]. In these theoretical studies, many permanent or extinct conditions for the models are obtained, however, the effects of the dispersal corridors on the permanence of the ecosystems have not been clearly revealed through these theoretical analysis.

Permanence is one of the basic concepts for bio-mathematical models on ecosystems, which is corresponding to the coexistence of biological species in ecosystems in large time scales [8]. In this paper, we first establish a discrete time model with two competitive species in patchy environment without dispersal corridor, and obtain sufficient conditions for the permanence of the model. Followed, a discrete time model with the same two competitive species in patchy environment but with a dispersal corridor is established, also sufficient conditions for the permanence of this model are obtained. After these, the sufficient conditions are compared and analyzed, the effects of the dispersal corridor are then revealed. That is, the effects of the dispersal corridors on the permanence of the competitive ecosystems are explored through theoretical analysis for the mathematical models.

It is well known that numerical simulation can be carried out to verify the theoretical results are whether well-posed or not. Two numerical examples are given in this paper. One numerical example shows that one of the biological species in one patch will extinct when the parameters of the model without dispersal corridor are specified as some certain values while the theoretical permanent conditions are not satisfied, and the other numerical example shows that both the two species are permanent when dispersal corridor is introduced while other parameters are the same as the first example and the theoretical permanent conditions with dispersal corridor are satisfied.

The paper is organized as follows: the theoretical permanent conditions for the competitive ecosystems with two species in patchy environment without or with dispersal corridor are obtained in section 2, respectively. In section 3, these sufficient conditions are compared and analyzed, the effects of the dispersal corridor are revealed. Numerical simulation is carried out in section 4.

2. PERMANENT RESULTS

2.1. Permanence without Dispersal Corridor

Suppose that there are two competitive biological species living in the same habitats, denote as x and y, respectively. Their habitats are separated in two patches, patch 1 and 2. Denote $x_i(n)$ (correspondingly, $y_i(n)$) as the density of the species x (y) at the n^{th} time step. It is supposed that the increasing of the densities of the species x (y) in these two patches is Logistic [9], we introduce the following model

$$\begin{cases} x_1(n+1) = x_1(n)\exp(r_{11} - a_{11}x_1(n) - b_{11}y_1(n)), \\ x_2(n+1) = x_2(n)\exp(r_{12} - a_{12}x_2(n) - b_{12}y_2(n)), \\ y_1(n+1) = y_1(n)\exp(r_{21} - a_{21}y_1(n) - b_{21}x_1(n)), \\ y_2(n+1) = y_2(n)\exp(r_{22} - a_{22}y_2(n) - b_{22}x_2(n)). \end{cases} \quad (1)$$

In model (1), r_{ij} ($i = 1,2, j = 1,2$) is the intrinsic increasing rate of species i in patch j; a_{1j} ($j = 1,2$) is the intra-specific competitive strength of species x in patch j; a_{2j} ($j = 1,2$) is the intra-specific competitive strength of species y in patch j; b_{1j} ($j = 1,2$) is the inter-specific competitive strength of

species y with x in patch j ; b_{2j} ($j=1,2$) is the inter-specific competitive strength of species x with y in patch j. All the parameters in model (1) are positive constants. The initial values of model (1) are

$$x_1(0)>0, x_2(0)>0, y_1(0)>0, y_2(0)>0\cdot \tag{2}$$

It is obvious that the solutions of model (1) with initial values (2) are positive that is consistent to the biological background.

Next we discuss the permanence of model (1) with initial values (2). we give the following lemma which can be easily obtained from Lemmas 1 and 2 in [10].

Lemma 2.1. Suppose that r,a are both positive constants, $\{x(n)\}_{n=1}^{\infty}$ is a positive sequence. If

$$x(n+1)\le x(n)\exp(r-ax(n)),$$

then

$$\limsup_{n\to\infty} x(n)\le \frac{1}{a}\exp(r-1);$$

If $\{x(n)\}_{n=1}^{\infty}$ satisfies

$$x(n+1)\ge x(n)\exp(r-ax(n))$$

and $\limsup_{n\to\infty} x(n)\le k$ (k is a positive constant), then

$$\liminf_{n\to\infty} x(n)\ge \frac{r}{a}\exp(r-ak)$$

provided that

$$\frac{1}{a}k>1.$$

For the simplicity, we denote

$$\begin{aligned} A_{11}&=\frac{1}{a_{11}}\exp(r_{11}-1),\ A_{12}=\frac{1}{a_{12}}\exp(r_{12}-1),\\ A_{21}&=\frac{1}{a_{21}}\exp(r_{21}-1),\ A_{22}=\frac{1}{a_{22}}\exp(r_{22}-1). \end{aligned} \tag{3}$$

The next theorem gives the sufficient conditions for the permanence of model (1) with initial values (2).

Theorem 2.2. If

$$r_{11}-b_{11}A_{21}>0, \tag{4}$$
$$r_{12}-b_{12}A_{22}>0, \tag{5}$$
$$r_{21}-b_{21}A_{11}>0, \tag{6}$$
$$r_{22}-b_{22}A_{12}>0, \tag{7}$$

then model (1) is permanent with initial values (2).

Proof. From the first equation of model (1),

$$x_1(n+1)\le x_1(n)\exp[r_{11}-a_{11}x_1(n)].$$

Hence, from Lemma 2.1,

$$\limsup_{n\to\infty} x_1(n)\le \frac{r}{a_{11}}\exp(r_{11}-1)=A_{11}. \tag{8}$$

Similarly,

$$\limsup_{n\to\infty} x_2(n)\le A_{12}, \tag{9}$$

$$\limsup_{n\to\infty} y_1(n)\le A_{21}, \tag{10}$$

$$\limsup_{n\to\infty} y_2(n)\le A_{22}. \tag{11}$$

From (10), there exists a positive integer N such that for any $\varepsilon>0$ one has for $n>N$

$$y_1(n)<A_{21}+\varepsilon. \tag{12}$$

From (12) and the second equation of model (1),

$$x_1(n+1)>x_1(n)\exp[r_{11}-b_{11}(A_{21}+\varepsilon)-a_{11}x_1(n)],$$

further, from Lemma 2.1,

$$\liminf_{n\to\infty} x_1(n)\ge \frac{r_{11}-b_{11}(A_{21}+\varepsilon)}{a_{11}}\exp[r_{11}-b_{11}(A_{21}+\varepsilon)-a_{11}A_{11}), \tag{13}$$

provided that

$$\frac{a_{11}}{r_{11}-b_{11}(A_{21}+\varepsilon)}A_{11}>1. \tag{14}$$

Denote

$$m_1^{(1)}=\frac{r_{11}-b_{11}A_{21}}{a_{11}}\exp(r_{11}-b_{11}A_{21}-a_{11}A_{11}),$$

$m_1^{(1)}>0$ is consequent from (4) and note that ε is arbitrarily given, from (13), (14),

$$\liminf_{n\to\infty} x_1(n)\ge m_1^{(1)}>0, \tag{15}$$

if

$$\frac{a_{11}}{r_{11}-b_{11}A_{21}}A_{11}>1, \tag{16}$$

holds. Again from (4), we get

$$\frac{a_{11}}{r_{11}-b_{11}A_{21}}A_{11}=\frac{\exp(r_{11}-1)}{r_{11}-b_{11}A_{21}}>\frac{r_{11}}{r_{11}-b_{11}A_{21}}>1.$$

That is, (16) is true if (4) holds. Moreover,

$$m_1^{(1)}<\frac{r_{11}-b_{11}A_{21}}{a_{11}}<\frac{r_{11}}{a_{11}}<\frac{1}{a_{11}}\exp(r_{11}-1)=A_{11},$$

since $r_{11}-b_{11}A_{21}-a_{11}A_{11}<0$ from (16). As a whole,

$$0<m_1^{(1)}\le\liminf_{n\to\infty} x_1(n)\le\limsup_{n\to\infty} x_1(n)\le A_{11}. \tag{17}$$

Similarly, there exists positive constants $m_1^{(2)},m_2^{(1)},m_2^{(2)}$ such that

$$0<m_1^{(2)}\le\liminf_{n\to\infty} x_2(n)\le\limsup_{n\to\infty} x_2(n)\le A_{12}, \tag{18}$$

$$0<m_2^{(1)}\le\liminf_{n\to\infty} y_1(n)\le\limsup_{n\to\infty} y_1(n)\le A_{21}, \tag{19}$$

$$0<m_2^{(2)}\le\liminf_{n\to\infty} y_2(n)\le\limsup_{n\to\infty} y_2(n)\le A_{22}. \tag{20}$$

Thus completes the proof of Theorem 2.2.

2.2 Permanence with Dispersal Corridor

In this part, we suppose that the two isolated patches in section 2.1 is now introduced one dispersal corridor. The species x can migrate between patch 1 and 2 with this corridor freely, but the species y can not disperse between these two patches, in this case, model (1) is rewritten as

$$\begin{cases} x_1(n+1) = x_1(n)\exp(r_{11} - a_{11}x_1(n) - b_{11}y_1(n) - d_{12}x_1(n) + d_{21}x_2(n)), \\ x_2(n+1) = x_2(n)\exp(r_{12} - a_{12}x_2(n) - b_{12}y_2(n) - d_{21}x_2(n) + d_{12}x_1(n)), \\ y_1(n+1) = y_1(n)\exp(r_{21} - a_{21}y_1(n) - b_{21}x_1(n) - D_{12}x_1(n) + D_{21}x_1(n)), \\ y_2(n+1) = y_2(n)\exp(r_{22} - a_{22}y_2(n) - b_{22}x_2(n) - D_{21}x_2(n) + D_{12}x_1(n)). \end{cases} \tag{21}$$

In model (21), the positive constant d_{ij} is the intra-specific competition strength due to the migration of species x from patch i to j ($i, j = 1, 2$), and the positive constant D_{ij} is the inter-specific competition strength due to the migration of species x from patch i to j ($i, j = 1, 2$). It is obvious that model (21) is the same as model (1) if $d_{ij} = D_{ij} = 0$, $i, j = 1, 2$.

Theorem 2.3. If

$$r_{11} - b_{11}A_{21} > 0, \tag{22}$$

$$r_{12} - b_{12}A_{22} > 0, \tag{23}$$

$$r_{21} - (b_{21} + D_{12})B_{11} > 0, \tag{24}$$

$$r_{22} - (b_{22} + D_{21})B_{12} > 0, \tag{25}$$

then model (21) is permanent with initial values (2), where

$$B_{11} = \frac{\exp(r_{11} - 1)}{a_{11} + d_{12}},\ B_{12} = \frac{\exp(r_{12} - 1)}{a_{12} + d_{21}}.$$

Proof. Considering the auxiliary system

$$\begin{cases} u(n+1) = u(n)\exp[r_{11} - (a_{11} + d_{12})u(n) - b_{11}v(n)], \\ v(n+1) = v(n)\exp[r_{21} - (b_{21} + D_{12})u(n) - a_{21}v(n)], \end{cases}$$

with initial values $u(0) > 0, v(0) > 0$. From the proof of Theorem 2.2, this system is permanent if (22) and (24) hold. Hence, there exists positive constants $m_1^{(1)}$ and $m_2^{(1)}$ such that

$$\liminf_{n\to\infty} u(n) \ge m_1^{(1)},\ \liminf_{n\to\infty} v(n) \ge m_2^{(1)}.$$

But for any positive integer n,

$$x_1(n+1) \ge u(n+1),\ y_1(n+1) \ge v(n+1),$$

hence,

$$\liminf_{n\to\infty} x_1(n) \ge m_1^{(1)},\ \liminf_{n\to\infty} y_1(n) \ge m_2^{(1)}. \tag{26}$$

Similarly,

$$\liminf_{n\to\infty} x_2(n) \ge m_1^{(2)},\ \liminf_{n\to\infty} y_2(n) \ge m_2^{(2)}, \tag{27}$$

where $m_1^{(1)}$ and $m_2^{(1)}$ are positive constants.

Next we prove the following to obtain the permanence of model (21) with initial values (2):

$$\begin{aligned} &\limsup_{n\to\infty} x_1(n) \le M_1^{(1)},\ \limsup_{n\to\infty} x_2(n) \le M_1^{(2)}, \\ &\limsup_{n\to\infty} y_1(n) \le M_2^{(1)},\ \limsup_{n\to\infty} y_2(n) \le M_2^{(2)}, \end{aligned} \tag{28}$$

where $M_i^{(j)}$ $(i, j = 1, 2)$ is a positive constant.

Suppose the contrary, there exists no $M_1^{(1)}$ such that

$$\limsup_{n\to\infty} x_1(n) \le M_1^{(1)},$$

therefore, there exists a subsequence $\{x_1(n_k)\}_{k=1}^{\infty}$ of $\{x_1(n)\}$, that satisfies

$$\lim_{k\to\infty} x_1(n_k) = +\infty.$$

Further, from the second equation and Lemma 2.1, there exists a subsequence $\{x_2(n_j)\}_{j=1}^{\infty}$ of $\{x_2(n)\}$, that satisfies

$$\lim_{j\to\infty} x_2(n_j) = +\infty. \tag{29}$$

Multiplying the first and the second equation of (21),

$$x_1(n+1)x_2(n+1) \le x_1(n)x_2(n)\exp[r_{11} + r_{12}a_{11}x_1(n) - a_{12}x_2(n)]$$

Note that the function $f(x, y) = xy\exp(r - ax - by)$ is bounded in the first quadrant, one has

$$\limsup_{n\to\infty} x_1(n+1)x_2(n+1)$$

exists finite. Note (29), $\lim_{j\to\infty} x_1(n_j) = 0$, which contradicts with the first inequality of (26). This contradiction shows that the first inequality of (28) holds. The other three inequalities of (28) can be similarly proven. Therefore, model (21) with initial values (2) is permanent from (26), (27) and (28). The proof of Theorem 2.3 is completed.

3. THE EFFECTS OF DISPERSAL CORRIDOR

The sufficient conditions for the permanence of model (1) and (21) have been obtained in section 2, respectively. In this section, we compare and analyze these two groups of sufficient conditions, that is, conditions (4)-(7) and (22)-(25).

The condition (4), (5) is the same as (22), (23), respectively. Condition (6), (7) can be written in the parameters of model (1) as

$$r_{21} - \frac{b_{21}}{a_{11}}\exp(r_{11} - 1) > 0, \tag{30}$$

and

$$r_{22} - \frac{b_{22}}{a_{12}}\exp(r_{12} - 1) > 0, \tag{31}$$

respectively. And condition (24), (25) is equivalent to

$$r_{21} - \frac{b_{21} + D_{12}}{a_{11} + d_{12}}\exp(r_{11} - 1) > 0, \tag{32}$$

and

$$r_{22} - \frac{b_{22} + D_{21}}{a_{12} + d_{21}}\exp(r_{12} - 1) > 0, \tag{33}$$

respectively. Comparing (30) with (32), and (31) with (33), the difference is that the terms d_{ij} and D_{ij} ($i, j = 1, 2$). Recalling model (1) and (21), these terms reflects the intra- or inter-specific competition strength due to the migration of the species x. It can be observed that the positive term d_{ij} in the denominator is helpful to satisfy both the conditions, (32) and (33), while the positive term D_{ij} in the numerator is a disadvantage both for the conditions, (32) and (33). Hence, the migration of the species x through the dispersal corridor is helpful for the survival of themselves, but the migration of the species x is a disadvantage for their competitors, the species y, since additional inter-specific competition is produced due to the dispersion of the species x via dispersal corridor.

But the values of $\frac{b_{21}+D_{12}}{a_{11}+d_{12}}$, $\frac{b_{22}+D_{21}}{a_{12}+d_{21}}$ are affected simultaneously by the terms d_{ij} and D_{ij} ($i, j=1,2$), therefore, the effects of dispersal corridor on the permanence of the competitive ecosystems are relevant to which competition strength due to the migration, the intra-specific competition or the inter-specific competition, is stronger.

4. NUMERICAL SIMULATION AND DISCUSSION

In this section, we carry out numerical simulation to confirm that the theoretical results of section 2 are well-posed and illustrate the explanation in section 3 for the effects of dispersal corridor on competitive ecosystems.

Example 4.1. Consider the following special case of model (1):

$$\begin{cases} x_1(n+1)=x_1(n)\exp(0.3-0.2x_1(n)-0.1y_1(n)), \\ x_2(n+1)=x_2(n)\exp(0.52-0.35x_2(n)-0.15y_2(n)), \\ y_1(n+1)=y_1(n)\exp(0.2-0.3y_1(n)-0.15x_1(n)), \\ y_2(n+1)=y_2(n)\exp(0.4-0.25y_2(n)-0.2x_2(n)). \end{cases} \quad (34)$$

In model (34), we have

$$r_{11}-b_{11}A_{21}=0.1502,\ r_{12}-b_{12}A_{22}=0.1907,$$
$$r_{21}-b_{21}A_{11}=-0.1724,\ r_{22}-b_{22}A_{12}=0.0464,$$

that is, condition (6) is not satisfied. Hence, model (34) is not permanent by Theorem 2.2, and it can be observed from Figure 1 that the species y in patch 1 will extinct.

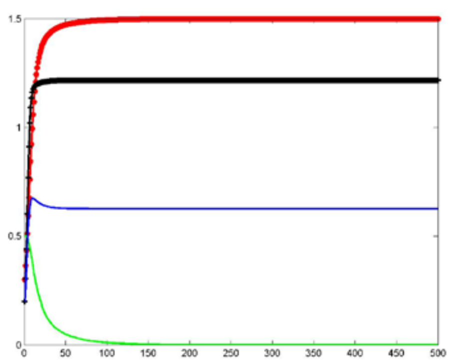

Figure 1. The variations of the population densities of model (34). The vertical axis is the population densities and the horizontal axis is the time. The red line is the population density of the species x in patch 1, the black line is the population density of the species x in patch 2, the green line is the population density of the species y in patch 1, and the blue line is the population density of the species y in patch 2. The species y in patch 1 will extinct, and model (34) is not permanent.

Example 4.2. Consider the following special case of model (21):

$$\begin{cases} x_1(n+1)=x_1(n)\exp(0.3-0.2x_1(n)-0.1y_1(n)-0.19x_1(n)), \\ x_2(n+1)=x_2(n)\exp(0.52-0.35x_2(n)-0.15y_2(n)), \\ y_1(n+1)=y_1(n)\exp(0.2-0.3y_1(n)-0.15x_1(n)-0.001x_1(n)), \\ y_2(n+1)=y_2(n)\exp(0.4-0.25y_2(n)-0.2x_2(n)). \end{cases} \quad (35)$$

In model (35), dispersal corridor is introduced for the species x , and the competition strengths produced by the migration are $d_{12}=0.19, D_{12}=0.001$ and $d_{21}=D_{21}=0$, while the other parameters are the same as model (34). The conditions (22)-(25) are

$$r_{11}-b_{11}A_{21}=0.1502,\ r_{12}-b_{12}A_{22}=0.1907,$$

$$r_{21}-(b_{21}+D_{12})B_{11}=0.0077,\ \ r_{22}-(b_{22}+D_{21})B_{12}=0.0464.$$

That is, the conditions (22)-(25) are all satisfied, hence, model (35) is permanent by Theorem 2.3, which is illustrated by Figure 2.

Model (34) together with model (35) shows the sufficient conditions of Theorem 2.2 and 2.3 are well-posed. Moreover, these two examples imply that the dispersal corridor is helpful for the permanence of the competitive ecosystems if the migration produces weaker inter-specific competition and decreases some stronger intra-specific competition at the same time, that is corresponding to the explanation of the last paragraph in section 3.

The effect of the dispersal corridor on competitive ecosystems is discussed with two species and two patches in this paper, It should be pointed out that Theorem 2.2 and 2.3 can be extended to competitive ecosystems with more species and patches.

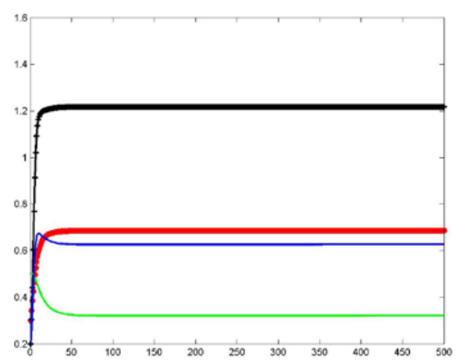

Figure 2. The variations of the population densities of model (35) that is permanent. The other aspects are the same as Figure 1.

5. ACKNOWLEDGEMENTS

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